

# PSYCHROMETRIC EQUATIONS: THEORY, DERIVATION AND APPLICATIONS IN HVAC

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## ABSTRACT

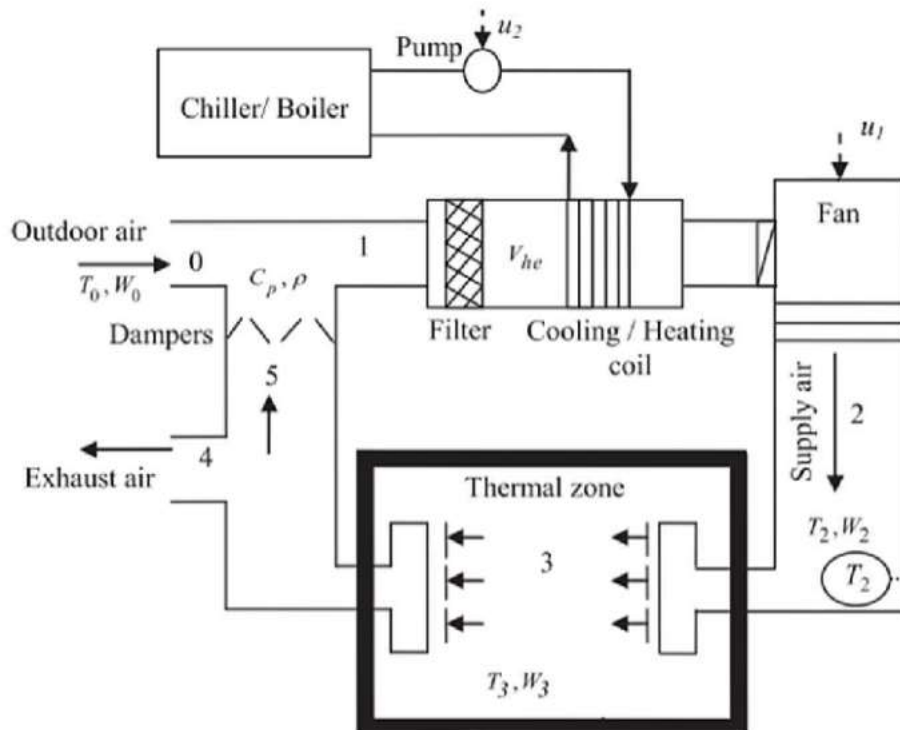
Psychrometrics deals with the thermodynamic properties of moist air and plays a vital role in HVAC engineering, meteorology, drying technology, and environmental control systems. This paper presents a comprehensive theoretical treatment of psychrometric equations, including the derivation of humidity ratio, relative humidity, dew point temperature, enthalpy of moist air, and wet-bulb temperature relations. Applications in engineering systems are also discussed.

**KEYWORDS:** Psychrometrics, Moist Air, Relative Humidity, Enthalpy, HVAC

## INTRODUCTION

Psychrometrics is the branch of thermodynamics that studies the properties and behavior of moist air. Accurate psychrometric

equations are essential for the analysis and design of air-conditioning systems, cooling towers, dryers, and atmospheric processes.



**Figure 1: Block diagram of a simple HVAC system (M. Gomez)**

Psychrometrics is a specialized branch of thermodynamics concerned with the study of the thermophysical properties and behavior of moist air. Since atmospheric air is a mixture of dry air and water vapor, its properties vary significantly with changes in temperature, pressure, and moisture content. Accurate

psychrometric relationships are therefore essential for understanding heat and mass transfer processes involving moist air under both steady and transient conditions.

$$P = Pa + Pv$$

P = total pressure



$P_a$  = partial pressure

$P_v$  = partial pressure

The analysis of moist air plays a fundamental role in the design and performance evaluation of a wide range of engineering systems. In particular, psychrometric equations form the theoretical foundation for heating, ventilation, and air-conditioning (HVAC) systems, where precise control of temperature, humidity, and air quality is required to ensure thermal comfort and energy efficiency. These equations are also critical in the design of cooling towers, where evaporative cooling processes dominate, and in industrial drying operations, where simultaneous heat and moisture transfer must be accurately modeled.

Derivation of Humidity Ratio:

The humidity ratio  $\omega$  is defined as the mass of water vapor per unit mass of dry air:

$$\omega = m_a/m_v$$

Using the ideal gas equation for dry air and water vapor,

$$m = RT/PV$$

$$\omega = P_a P_v / R_v R_a$$

Since  $R_a/R_v = 0.622$

$$\omega = P_a / (0.622) P_v$$

Beyond engineering applications, psychrometrics is of significant importance in meteorology, climatology, and environmental science. Atmospheric processes such as cloud formation, precipitation, and weather prediction rely heavily on the thermodynamic behavior of moist air. The use of psychrometric principles enables the evaluation of atmospheric stability, humidity variations, and energy exchange between the Earth's surface and the atmosphere.

This study focuses on the fundamental psychrometric properties of moist air and the governing equations that describe them. Key parameters such as humidity ratio, relative humidity, dew point temperature, wet-bulb temperature, and enthalpy are examined in detail, along with their interrelationships. A rigorous understanding of these psychrometric equations is essential for accurate system analysis, optimal design, and efficient operation of thermal and environmental control systems.

## 2. Composition of Moist Air

Moist air is a homogeneous mixture primarily composed of dry air and water vapor, with minor constituents such as carbon dioxide, argon, and other trace gases. In psychrometric analysis, moist air is typically modeled as a binary mixture of dry air and water vapor, both of which are assumed to behave as ideal gases under standard atmospheric conditions. According to Dalton's law of partial pressures, the total pressure of moist air is equal to the sum of the partial pressure of dry air and the partial pressure of water vapor (Çengel & Boles, 2019). While the composition of dry air remains nearly constant, the water vapor content varies significantly with temperature and pressure, leading to changes in fundamental psychrometric properties such as humidity ratio, relative humidity, and dew point temperature (ASHRAE, 2017).

A clear understanding of the composition of moist air is therefore essential for accurate analysis of heat and mass transfer processes in HVAC systems, drying technologies, and atmospheric applications (Moran et al., 2018). Moist air is a mixture of dry air and water vapor. It is commonly treated as an ideal gas mixture.

## Pressure

$$P = P_a + P_v$$

where  $P_a$  is the partial pressure of dry air and  $P_v$  is the partial pressure of water vapor.

### 3. Humidity Ratio

The humidity ratio ( $\omega$ ) is defined as the mass of water vapor per unit mass of dry air:

$$\omega = 0.622 * (P_v / (P - P_v))$$

### 4. Relative Humidity

Relative humidity ( $\phi$ ) is the ratio of the actual vapor pressure to the saturation vapor pressure at the same temperature:

$$\phi = P_v / P_{vs}$$

### 5. Dew Point Temperature

The dew point temperature is the temperature at which moist air becomes saturated when cooled at constant pressure. It is obtained by equating vapor pressure to saturation pressure:

$$P_v = P_{vs}(T_{dp})$$

### 6. Enthalpy of Moist Air

The specific enthalpy of moist air per kg of dry air is given by:

$$h = 1.005T + \omega(2500 + 1.88T)$$

where T is in °C and h is in kJ/kg dry air.

### 7. Wet-Bulb Temperature

The wet-bulb temperature is determined using the energy balance between sensible heat loss and latent heat gain:

$$h = h_s(T_{wb})$$

## 9. Applications

Psychrometric equations are used in HVAC system design, agricultural drying, thermal comfort analysis, and meteorology. HVAC systems are essential in residential, commercial, and industrial buildings. Proper design requires accurate mathematical modeling of thermal loads, air properties, and equipment performance.

### Heating Load Calculation

The total heating load is calculated as:

$$Q_h = UA \int (T_i - T_o)$$

Where  $Q_h$  is heating load (W), U is overall heat transfer coefficient, A is area (m<sup>2</sup>),  $T_i$  and  $T_o$  are indoor and outdoor temperatures.

### Cooling Load Calculation

Total cooling load consists of sensible and latent loads:



$$Q_{-c} = Q \int f_{-s} + \int Q_{-l}$$

Sensible cooling load:

$$Q_{-s} = \dot{m}_{-a} \int c_{-p} (T_{-i} - T_{-s})$$

Latent cooling load:

$$Q_{-l} = \dot{m}_{-a} \int h_f \iint (p, g) (\omega_{-i} - \omega_{-s})$$

Airflow Rate Requirement

The required mass flow rate of air is:

$$\dot{m}_{-a} = Q_{-s} / \int [c_{-p} (T_{-i} - T_{-s})]$$

Volumetric flow rate:

$$V = \dot{m}_{-a} / \rho$$

Psychrometric Relations in HVAC

Humidity ratio:

$$\omega = 0.622 (P_{-v} / (P - P_{-v}))$$

Relative humidity:

$$\phi = P_{-v} / P_{-vs}$$

Enthalpy of moist air:

$$h = 1.005T + \omega(2500 + 1.88T)$$

Duct Design Equations

Continuity equation:

$$V = A v$$

Pressure loss due to friction (Darcy–Weisbach):

$$\Delta P = f (L/D) (\rho v^2 / 2)$$

Cooling capacity:

$$Q_{-L} = \dot{m}_{-r} (h_{-1} - h_{-4})$$

Coefficient of performance:

$$COP = Q_{-L} / W_{-c}$$

HVAC system design equations provide a scientific basis for selecting equipment, designing ductwork, and ensuring occupant comfort with minimal energy consumption.

## CONCLUSION

Psychrometric equations form the backbone of moist air analysis. Their theoretical understanding is essential for accurate modeling and efficient system design.

Psychrometric analysis provides a fundamental framework for understanding the thermodynamic behavior of moist air and its role in coupled heat and mass transfer processes. The formulation and derivation of essential psychrometric relationships—such as humidity ratio, relative humidity, dew point temperature, wet-bulb temperature, and specific enthalpy—constitute the theoretical foundation required for accurate prediction and control of moist air behavior under varying environmental conditions (Çengel & Boles, 2019; Moran et al., 2018). These equations are indispensable in the analysis and design of heating,

ventilation, and air-conditioning (HVAC) systems, drying operations, and environmental control applications, where precise regulation of temperature and humidity is critical (ASHRAE, 2017).

Summary of Equations

$$Qh = UA(Ti - To)$$

$$Qc = Qs + Ql$$

$$Qs = \dot{m}'_{-a} c_p (Ti - Ts)$$

$$Ql = \dot{m}'_{-a} h_f g (\omega_i - \omega_s)$$

Moreover, the application of psychrometric principles extends to meteorology and atmospheric sciences, underscoring their broader scientific and practical relevance. A rigorous understanding of psychrometric equations enables engineers and researchers to optimize system performance, enhance energy efficiency, and improve indoor environmental quality, thereby contributing to sustainable and resilient engineering solutions (McQuiston et al., 2005). As such, psychrometrics remains a vital and enduring component of thermodynamic analysis, effectively bridging theoretical principles with real-world engineering applications.

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