ISSN (Online): 2455-3662
EPRA International Journal of Multidisciplinary Research (IJMR) - Peer Reviewed Journal
Volume: 12| Issue: 3| March2026|| Journal DOI: 10.36713/epra2013 || SJIF Impact Factor 2026: 8.753 || ISI Value: 1.188

DERIVATION OF CONNECTED CERTIFIED DOMINATION EDGE-CRITICAL GRAPHS

Megha Chandraprakash Gaikwad[1]
[1]*Assistant Professor, Shri Sheth Muralidharji Mansingaka Arts, science and commerce College, Pachora , Dist. Jalgaon*

ABSTRACT
By evaluating the dynamics of the CCDN when edges are removed from a graph, this paper focuses on the relatively new idea of certified domination in domination theory, which was first introduced in 2020. The paper provides characterizations, propositions, and structural properties for both classes of graphs. To be specific, we focus on derivation of the CCDN edge critical graphs for categorizing generalized orientation.
KEY WORDS: *Connected Certified Domination, Edge-Critical Graphs, and Edge Removal Criticality.*

A Connected Isolate-Free Graph
A connected isolate-free graph is a concept from Graph Theory that combines two properties of a graph.

Connected Graph
A graph is connected if there is a path between every pair of vertices.
This means you can start at any vertex and reach any other vertex by following edges.

Isolate-Free Graph
An isolated vertex is a vertex with degree 0. A graph is isolate-free if every vertex has at least one edge. $deg(v) \geq 1$ for every vertex v

Certified Dominating Set (CDS)
A set $C \subseteq VZ$ is called a certified dominating set if for every vertex $u \in C$:

$$|NZ(u) \cap (VZ \setminus C)| = 0 or\ |NZ(u) \cap (VZ \setminus C)| \geq 2$$

This means each vertex in CCC:

- Either has no neighbour outside C, or
- Has at least two neighbours outside C.

The minimum size of such a set is the certified domination number: $\gamma cer(Z)$

Connected Certified Dominating Set (CCDS):
A set CCC is a connected certified dominating set if:

1. CCC is a certified dominating set
2. the induced subgraph $Z[C]$ is connected

The minimum size of such a set is called the $\gamma ccer(Z)$
Edge-Critical Graphs:
A graph Z is connected certified domination edge-critical if:

$$\gamma ccer(Z-e) > \gamma ccer(Z) \forall e \in EZ$$

Deleting any edge increases the connected certified domination number.

Crucial Observation
Let C be a minimum CCDS.

For each $u \in Cu \setminus in\ Cu \in C$:

$|epn(u,C)| \geq 1$ Where

$$epn(u,C) = pn(u,C) \cap (V-C)$$

This means every vertex in the CCDS must have at least one external private neighbour.
This condition ensures liminality of the CCDS.

Derivation Using Structural Properties:
Graphs with Diameter ≤ 2 and Minimum Degree 1:

$$diam(Z) \leq 2$$

$$\delta(Z) = 1$$

Then there exists at least one leaf vertex**.**

For diameter 2:

$$\gamma ccer(Z) = 1$$

Because a central vertex dominates the graph**.**

Let edge $e = uv$

After removing e:

- the leaf becomes isolated or poorly dominated
- Domination conditions fail

$$\gamma ccer(Z-e) > \gamma ccer(Z)$$

Effect of Strong Support Vertices:
A strong support vertex is a vertex adjacent to two or more leaves**.**

Let

$$u \in S2(Z)$$

And v be a leaf.
Since: $N(v) = \{u\}$

Vertex u must belong to every CCDS.

If edge uv is deleted:

- vertex v becomes isolated
- domination condition fails

Thus $\gamma ccer(Z - e) = \infty$

Therefore Z is edge-critical.

Edge-Critical Condition Using Neighbourhoods:

Let

$$C = \gamma ccer(Z) - set$$

And edge $e = uv$

Such that;

$u \in C, v \in/C$ and $| N(v) \cap C |= 1$

Then vertex v is dominated by only one vertex of C.

Deleting uv:

- removes the only domination of v
- forces inclusion of additional vertices in CCDS

Hence $\gamma ccer(Z - e) > \gamma ccer(Z)$

Thus the graph becomes edge-critical.

Characterization Theorem:

The main result states:

$Z\ is\ [\gamma ccer]e - critical$ if and only if $Z \in T$

Where T is the family of trees satisfying:

- every vertex is either
  - a leaf, or
  - has degree ≥ 3

Final Result
A graph is Connected Certified Domination Edge-Critical if: $\gamma ccer(Z - e) > \gamma ccer(Z)$
And structurally this occurs exactly for graphs in family T consisting of star-like trees with strong support vertices and leaves.

Significance and Contribution
The study extends the theory of certified domination, a recently introduced concept in graph domination theory. Investigating how graph parameters change under edge deletion is an important direction in domination theory and network reliability analysis.
The main contribution of the paper is the characterization of connected certified domination edge-critical graphs and the identification of conditions that guarantee edge stability.

Conclusion
Overall, the paper provides a valuable contribution to domination theory, particularly in the study of the connected certified domination number under edge deletion. The characterization of edge-critical graphs and identification of stable graph classes enrich the theoretical understanding of domination parameters in graphs.
Despite minor presentation issues, the results are mathematically sound and open promising avenues for further research.

REFERENCES

Lone, A. I., & Goswami, V. (2023). Connected certified domination edge critical and stable graphs. Acta Universitatis Sapientiae, Informatica, 15(1), 25–37. https://doi.org/10.2478/ausi-2023-0003

*Dettlaff, M., Lemańska, M., Topp, J., Ziemann, R., & Żyliński, P. (2020).
Certified domination.
AKCE International Journal of Graphs and Combinatorics, 17(1), 86–97.*

*Hamja, J. (2023).
Certified perfect domination in graphs.
European Journal of Pure and Applied Mathematics.*

*Hamja, J., Laja, A., Copel, H., Amiruddin-Rajik, B., & Mohammad, N. (2025).
On certified perfect domination in graphs.
European Journal of Pure and Applied Mathematics.*

*Saroja, M. D., & Aneesh, R. (2023).
Connected certified domination in the middle graph of certain graphs.
Journal for ReAttach Therapy and Developmental Diversities.*

*Sharada, B., Ashwini, B., & Nayaka, S. R. (2016).
Perfect domination edge subdivision critical and stable graphs.
International Journal of Mathematics and Its Applications.*

*Kuriakose, R., & Parvathy, K. S. (2023).
Stability of domination in graphs.
Ratio Mathematica.*

*Hormillada, C. C. (2019).
Characterizations of connected perfect domination in graphs.
International Journal for Research in Mathematics and Statistics.*