ISSN (Online): 2455-3662
EPRA International Journal of Multidisciplinary Research (IJMR) - Peer Reviewed Journal
Volume: 12| Issue: 5| May 2026|| Journal DOI: 10.36713/epra2013 ||SJIF Impact Factor 2026: 8.753 || ISI Value: 1.188

DEVELOPING A THEORY-BASED METACOGNITIVE INSTRUCTIONAL DESIGN FOR GRADE 6 MATHEMATICAL PROBLEM-SOLVING

Jay L. Galimpin, MAEd; **Victoria Jaum,** MA
College of Education, Holy Name University

Article DOI: https://doi.org/10.36713/epra27611
DOI No: 10.36713/epra27611

ABSTRACT

Mathematical problem-solving requires learners to interpret task conditions, select appropriate strategies, monitor solution processes, and evaluate the reasonableness of answers. However, classroom instruction often emphasizes procedural accuracy without sufficiently making learners' thinking processes explicit. This design-and-development study developed a theory-based metacognitive instructional design for Grade 6 mathematical problem-solving, specifically in finding the area of composite figures and solving routine and non-routine problems involving composite figures. The study translated Divinagracia's Metacognitive Theory into a classroom-ready instructional sequence using four development phases: contextual analysis, theory-to-practice mapping, expert review and revision, and implementation planning. The design was anchored on seven metacognitive domains: knowledge of the typology of mathematical problems, knowledge of the nature of mathematical problems, awareness of mathematical knowledge and thinking, knowledge of personal strengths, knowledge of problem-solving emotions and attitudes, knowledge of thinking associated with bodily motion experiences, and metacognitive solution qualities. Expert review informed revisions that simplified learner prompts, improved time allocation, reduced cognitive load, and embedded metacognitive processes through guided questioning and observable learner actions. The resulting design integrated shape recognition, routine and non-routine problem analysis, guided formula application, self-questioning, hands-on measurement, real-life valuing, and structured reflection. The study contributes a context-sensitive instructional model for operationalizing metacognition in elementary mathematics by aligning lesson phases with planning, monitoring, regulation, affective engagement, embodied cognition, and evaluation.

KEYWORDS: *metacognition, instructional design, mathematical problem-solving, composite figures, Grade 6 mathematics*

INTRODUCTION

Mathematical problem-solving is a fundamental goal of mathematics education because it enables learners to use mathematical ideas in interpreting, representing, and resolving both familiar and unfamiliar situations. In the elementary grades, problem-solving is especially important because learners begin to move from direct procedural application toward more strategic forms of reasoning. When learners solve mathematical problems, they are expected not only to recall formulas or perform computations but also to understand the task, identify relevant information, select appropriate strategies, monitor their solution process, and evaluate the reasonableness of their answers. These demands are intensified in non-routine problems, where the solution path is not immediately apparent and where learners must regulate their thinking more deliberately.

Despite its importance, mathematical problem-solving remains difficult for many learners. A common classroom challenge is that students may know a formula but may not know when, why, or how to use it in a particular problem situation. This difficulty becomes evident in geometry and measurement tasks such as composite-area problem-solving, where learners must recognize component figures, decompose a larger figure into manageable parts, connect each part to an appropriate formula, and combine partial results accurately. In such tasks, success depends not only on procedural knowledge but also on learners' capacity to plan, monitor, adjust, and evaluate their thinking.

Metacognition provides a strong instructional lens for addressing this challenge. Generally understood as awareness and regulation of one's own thinking, metacognition helps learners examine how they approach a task, what strategies they use, where they encounter difficulty, and how they may improve or revise their solution process (Flavell, 1979). In mathematical problem-solving, metacognition allows learners to move beyond mechanical computation toward strategic and reflective engagement. It supports learners in asking questions such as: What kind of problem is this? What information is given? What do I need to find? What strategy should I try? Is my answer reasonable? What should I change if my solution does not work?

Previous research has shown that metacognitive instruction can support mathematical learning by strengthening learners' planning, monitoring, and evaluation processes (Jacobse & Harskamp, 2009; Lee et al., 2014; Okumuş & Öztürk, 2024). Studies have also linked metacognition with mathematics achievement and problem-solving success (Schneider & Artelt,

ISSN (Online): 2455-3662
EPRA International Journal of Multidisciplinary Research (IJMR) - Peer Reviewed Journal
Volume: 12| Issue: 5| May 2026|| Journal DOI: 10.36713/epra2013 ||SJIF Impact Factor 2026: 8.753 || ISI Value: 1.188

2010; Xie et al., 2024). However, the instructional application of metacognition remains a continuing concern. In many classrooms, metacognition is encouraged through general reminders such as "think carefully," "check your work," or "explain your answer." Although such prompts may be useful, they may not be sufficient to develop learners' metacognitive capacity unless they are embedded in a systematic instructional design that makes thinking processes explicit, observable, and teachable.

A further limitation is that metacognitive instruction is often reduced to a narrow cycle of planning, monitoring, and evaluation. While these processes are central, mathematical problem-solving also involves other dimensions of learner experience. Students must recognize whether a problem is routine or non-routine, understand the structure and nature of the task, become aware of their own strengths and limitations, manage emotions such as uncertainty or frustration, and use concrete or embodied experiences to support abstract reasoning. These dimensions are particularly relevant for elementary learners who are still developing their ability to connect visual, physical, affective, and symbolic aspects of mathematical thinking.

The present study addressed this need by developing a theory-based metacognitive instructional design for Grade 6 mathematical problem-solving. The design was grounded in Divinagracia's Metacognitive Theory, a Philippine-context theory that conceptualizes successful mathematical problem-solving as an integrated metacognitive phenomenon involving task-related knowledge, mathematical knowledge and thinking, personal strengths, affective and bodily motion experiences, and solution-regulating qualities (Divinagracia, 2018, 2022). Instead of treating metacognition as an additional reflection exercise after problem-solving, this study embedded metacognitive processes across the full instructional sequence.

The novelty of this study lies in the operational translation of a Philippine-context metacognitive theory into a classroom-ready instructional design for elementary mathematics. While prior literature has established the importance of metacognition in mathematics learning, fewer studies show how a multidimensional metacognitive theory can be transformed into specific lesson phases, teacher prompts, learner actions, and observable indicators for Grade 6 problem-solving instruction. This study therefore contributes not by testing a new statistical relationship, but by producing a theoretically grounded and practically usable instructional model that may guide future intervention and effectiveness studies.

The study focused on Grade 6 composite-area problem-solving because this content provides a meaningful context for operationalizing metacognition. Learners must identify component shapes, interpret problem conditions, apply area formulas, solve routine and non-routine problems, and check the accuracy of their answers. These tasks require the coordination of mathematical knowledge and self-regulatory thinking. By developing an instructional design aligned with metacognitive domains, the study sought to provide a classroom-applicable model for helping learners engage more deliberately and reflectively in mathematical problem-solving.

The study was guided by the following research question: What theory-based metacognitive instructional design can be developed for Grade 6 mathematical problem-solving?

THEORETICAL FRAMEWORK

Metacognition in Mathematical Problem-Solving

Metacognition refers to learners' awareness and regulation of their own thinking processes (Flavell, 1979). In mathematical problem-solving, metacognition enables learners to examine what a problem asks, determine what information is available, identify suitable mathematical concepts, monitor solution attempts, recognize errors, and evaluate the adequacy of final answers. This regulatory function is particularly important in non-routine tasks, where learners cannot rely solely on familiar algorithms or rehearsed procedures.

Research in mathematics education has repeatedly linked metacognition to mathematical achievement and problem-solving success. Schneider and Artelt (2010) emphasized that metacognition is a meaningful factor in mathematics learning, while Xie et al. (2024) reported a positive relationship between metacognition and mathematics achievement across educational levels. Other studies have shown that metacognitive instruction can strengthen learners' ability to plan, monitor, and evaluate mathematical solutions (Jacobse & Harskamp, 2009; Lee et al., 2014; Okumuş & Öztürk, 2024). These findings suggest that instruction should not only transmit mathematical content but also explicitly support learners in regulating how they use mathematical knowledge.

Divinagracia's Metacognitive Theory

This study was anchored on Divinagracia's Metacognitive Theory in successful mathematical problem-solving. The theory identifies metacognitive processes that shape learners' ability to arrive at successful mathematical solutions. It emphasizes that successful problem-solving involves more than cognitive knowledge; it also includes awareness of task demands, affective dispositions, bodily or motion-related thinking, and solution-regulating strategies (Divinagracia, 2018, 2022).

The theory is organized around seven interconnected domains. The first domain, metacognitive knowledge of the typology of mathematical problems, refers to learners' awareness of whether a task is routine or non-routine and whether it requires basic or more complex solution approaches. The second domain, metacognitive knowledge of the nature of mathematical problems, concerns learners' understanding of the problem structure, including the given information, required answer, and underlying mathematical relationships. The third domain, metacognitive awareness of mathematical knowledge and thinking, refers to learners' awareness of the mathematical concepts, formulas, and procedures needed for solution. The fourth domain, metacognitive knowledge of personal strengths, involves learners' awareness of their capabilities, limitations, and confidence during problem-solving. The fifth domain, metacognitive knowledge of problem-solving emotions and attitudes, recognizes that persistence, anxiety, interest, and sense of responsibility influence engagement with mathematical tasks. The sixth domain, metacognitive knowledge of thinking associated with bodily motion experiences, highlights the role of physical actions, gestures, measurement, tracing, pointing, and embodied engagement in supporting mathematical understanding. Finally, metacognitive solution qualities involve regulatory strategies such as planning, monitoring, checking, revising, evaluating, and controlling time and direction during the solution process.

These seven domains provided the design architecture of the present instructional intervention. Each domain was translated into a classroom activity, learner behavior, and metacognitive indicator to ensure that the theory was operationalized in observable instructional practice.

Design Thinking as an Instructional Support

The instructional activities also drew from general principles of design thinking. In educational contexts, design thinking supports learner-centered inquiry, problem analysis, ideation, testing, and reflection (Brown, 2008; Pande & Bharathi, 2020). For this study, design thinking was not used as a separate theoretical model but as a practical design orientation that helped structure learner engagement with mathematical problems. The design encouraged learners to identify the problem, analyze its structure, try strategies, monitor progress, and reflect on solution quality. This complemented the metacognitive orientation of the intervention by emphasizing problem interpretation, strategy selection, iterative improvement, and reflective evaluation.

METHOD

Research Design

This study used a design-and-development research approach to produce a theory-based metacognitive instructional design for Grade 6 mathematical problem-solving. Design-and-development research is appropriate when the purpose is to create, refine, and document an educational intervention based on theoretical and contextual considerations (van den Akker et al., 2006). In this study, the primary research output was the instructional design itself: a structured sequence of mathematics learning activities aligned with Divinagracia's seven metacognitive domains.

Although the instructional design formed part of a larger quantitative thesis project, the present article focuses only on the development component corresponding to the first research question. Therefore, the results reported here are limited to the structure, theoretical alignment, validation-informed refinement, and instructional logic of the developed design. Effectiveness outcomes are not the main focus of this article.

The development process followed four phases. The first phase was contextual analysis, which involved identifying the Grade 6 mathematics competency, the problem-solving demands of composite-area tasks, and the need to support learners' metacognitive engagement. The second phase was theory-to-practice mapping, in which the seven domains of Divinagracia's Metacognitive Theory were translated into instructional activities, expected learner actions, and metacognitive indicators. The third phase was expert review and revision, during which the lesson design was evaluated for content accuracy, theoretical alignment, developmental appropriateness, clarity, and classroom feasibility. The fourth phase was implementation planning, which involved preparing teacher guidance, lesson materials, and fidelity safeguards to support consistent classroom enactment.

Instructional Context

The instructional design was developed for Grade 6 learners in a mathematics lesson on composite area. The content focused on composite figures formed by two or more plane figures, including triangles, squares, rectangles, circles, and semicircles. The lesson addressed two Grade 6 competencies: finding the area of composite figures and solving routine and non-routine problems involving the area of composite figures. These competencies were selected because they require learners to connect conceptual understanding, procedural knowledge, visual-spatial analysis, and problem-solving regulation.

Composite-area problem-solving was considered suitable for metacognitive instructional design for three reasons. First, learners must recognize the component figures within a composite shape before applying formulas. Second, learners must monitor whether their selected formulas and computations match the given problem. Third, routine and non-routine tasks require different levels of strategic flexibility, making problem typology an important metacognitive concern.

Design Basis

The design process began by mapping Divinagracia's seven metacognitive domains onto the natural flow of a Grade 6 mathematics lesson. The goal was to avoid adding metacognition as an isolated activity and instead embed it throughout the lesson sequence. Each domain was translated into four instructional elements: (a) an instructional activity, (b) expected learner action, (c) intended learner development, and (d) metacognitive indicator.

The mapping process followed this logic. The typology and nature of mathematical problems were placed at the beginning of the lesson because learners first need to identify what kind of problem they are solving and what the problem requires. Mathematical knowledge and thinking were placed during formula recall and guided examples because learners need to connect prior knowledge with the problem situation. Personal strengths and bodily motion experiences were embedded in guided and hands-on practice because learners need opportunities to monitor their understanding and use concrete experiences to

ISSN (Online): 2455-3662
EPRA International Journal of Multidisciplinary Research (IJMR) - Peer Reviewed Journal
Volume: 12| Issue: 5| May 2026|| Journal DOI: 10.36713/epra2013 ||SJIF Impact Factor 2026: 8.753 || ISI Value: 1.188

support abstraction. Emotions and attitudes were addressed through valuing activities that linked accuracy with real-life consequences. Solution qualities were emphasized during evaluation and closure, where learners checked, justified, revised, and reflected on their solutions.

The theory-to-practice mapping was guided by three design principles. First, each activity had to correspond directly to one metacognitive domain. Second, each domain had to be represented through learner actions that could be observed during instruction. Third, the sequence had to follow the natural rhythm of mathematical problem-solving: understanding the task before solving, monitoring during solving, and evaluating after solving. These principles ensured that the design remained both theoretically coherent and pedagogically feasible.

Development and Validation Process

The instructional design was developed as a structured lesson sequence and then reviewed by a licensed elementary mathematics teacher and a mathematics education expert. The review focused on content accuracy, alignment with the Grade 6 competency, integration of metacognitive constructs, appropriateness of strategies, lesson coherence, and developmental suitability for Grade 6 learners.

Expert feedback led to several revisions. Metacognitive prompts were simplified to make them more understandable for Grade 6 learners. Time allocation was improved to make the lesson feasible within classroom constraints. The instructional flow was streamlined to reduce cognitive load. Instead of requiring learners to complete rigid metacognitive forms, metacognitive processes were embedded through guided questioning, verbalization, self-checking, and reflection. The activity sequence was also refined to progress from visual identification and decomposition of shapes to guided computation and evaluative checking.

The revision process produced three major refinements. First, the language of metacognitive prompts was made more age-appropriate so that learners could respond without being burdened by technical terminology. Second, activities were reordered to move from concrete and visual experiences toward symbolic computation and reflective evaluation. Third, the teacher's role was clarified as a facilitator of thinking through prompts, pauses, and guided questions rather than as a provider of procedural steps only. These refinements strengthened the alignment between metacognitive theory and classroom implementation.

Implementation Safeguards

The design included safeguards to support fidelity of implementation. The researcher provided a theory orientation to the teacher-facilitator, prepared detailed lesson plans, gave structured guidance, and used a fidelity checklist aligned with metacognitive strategy awareness, regulation, and flexibility. The researcher designed the intervention but did not directly facilitate the instructional sessions to minimize conflict of interest and preserve objectivity. A licensed and experienced Grade 6 mathematics teacher facilitated the intervention.

Ethical safeguards were observed in the larger study context. Permissions were secured from relevant institutional authorities, and the study followed procedures for informed consent, learner assent, confidentiality, anonymity, and voluntary participation.

RESULTS

The study produced a theory-based metacognitive instructional design for Grade 6 composite-area problem-solving. The design translated Divinagracia's seven metacognitive domains into a lesson sequence composed of classroom activities, expected learner actions, and metacognitive indicators. The final design was the product of contextual analysis, theory-to-practice mapping, expert-informed revision, and implementation planning.

The results are presented in three parts. First, the final instructional design is shown as a instructional matrix. Second, the structure of the design is explained according to how each metacognitive domain was operationalized in classroom practice. Third, the validation-informed refinements are summarized to show how expert review shaped the final design. Table 1 presents the developed instructional design.

Metacognitive domain	Instructional translation	Expected learner action	Metacognitive indicator
Knowledge of the typology of mathematical problems	Shape recognition drill and picture analysis	Identify geometric figures, distinguish simple and composite figures, and classify tasks as familiar, unfamiliar, routine, or non-routine	Task identification and planning
Knowledge of the nature of mathematical problems	Guided discussion of routine and non-routine problems; concept formation on area	Determine given information, missing information, problem structure, and expected output	Problem framing and interpretation
Awareness of mathematical knowledge and thinking	Formula recall, teacher modeling, and guided solution of composite figures	Select and apply appropriate formulas; connect prior knowledge with new problem contexts	Strategy awareness and monitoring

ISSN (Online): 2455-3662
EPRA International Journal of Multidisciplinary Research (IJMR) - Peer Reviewed Journal
Volume: 12| Issue: 5| May 2026|| Journal DOI: 10.36713/epra2013 ||SJIF Impact Factor 2026: 8.753 || ISI Value: 1.188

Knowledge of personal strengths	Guided self-questioning during problem-solving	Verbalize solution steps, identify confusion, seek clarification, and justify chosen methods	Self-monitoring and regulation
Knowledge of thinking associated with bodily motion experiences	Hands-on measurement using rulers; tracing, pointing, and decomposing figures	Measure real objects, trace dimensions, point to component shapes, and physically support computation	Embodied cognition and cognitive support
Knowledge of problem-solving emotions and attitudes	Valuing activity using real-life applications such as tile installation	Reflect on the importance of accuracy, responsibility, and persistence in solving mathematical problems	Affective regulation
Metacognitive solution qualities	Planning, midway checking, evaluation tasks, and closure reflection	Write solution plans, pause to check progress, revise errors, justify answers, and reflect on strategies	Evaluation, regulation, and solution control

Structure of the Developed Instructional Design

The instructional design followed a progression from task recognition to reflective evaluation. The first phase introduced composite figures through shape recognition and picture analysis. This activity required learners to identify component shapes and distinguish simple figures from composite figures. At the metacognitive level, the activity supported task identification because learners were guided to recognize the nature of the mathematical object before solving.

The second phase addressed the nature of mathematical problems. Learners discussed the difference between routine and non-routine problems and identified what information was given, what information was missing, and what the problem required. This phase was intended to prevent premature computation by helping learners frame the problem before selecting a strategy.

The third phase focused on mathematical knowledge and thinking. Through formula recall and guided examples, learners connected prior knowledge of area formulas with composite figures. This phase emphasized strategic awareness by asking learners to consider why a particular formula or decomposition strategy was appropriate for the problem.

The fourth phase embedded guided self-questioning. Learners were prompted to explain their reasoning, identify points of confusion, and justify their methods. This activity supported metacognitive regulation because learners were expected to monitor their understanding and adjust their approach when necessary.

The fifth phase introduced bodily motion experiences through hands-on measurement, tracing, pointing, and decomposition of figures. Learners used rulers to measure objects and physically represented parts of composite figures. This activity translated the theory's embodied dimension into classroom practice by connecting abstract mathematical concepts with concrete physical engagement.

The sixth phase addressed emotions and attitudes through a valuing activity. Learners discussed real-life contexts in which accurate area computation matters, such as tile installation. This phase supported affective regulation by encouraging learners to associate mathematical accuracy with responsibility and practical consequences.

The final phase emphasized metacognitive solution qualities. Learners wrote solution plans, paused during problem-solving to check their progress, evaluated the reasonableness of their answers, revised errors, and reflected on their strategies. This phase integrated planning, monitoring, evaluation, and regulation into a complete metacognitive cycle.

Three-Day Instructional Sequence

The developed design was organized into a three-day instructional sequence. On the first day, the lesson focused on introducing composite figures, recognizing component shapes, classifying problems, and planning solution steps. Learners were guided to verbalize their thinking before solving. On the second day, the lesson emphasized guided practice, monitoring, and strategy adjustment. Learners decomposed figures and solved problems while responding to metacognitive prompts. The teacher facilitated midway pauses to help learners revisit their plans and check their progress. On the third day, the lesson focused on solving routine and non-routine problems independently. Learners justified their solutions, checked the reasonableness of answers, and reflected on the strategies they used.

This sequence shows that the design was not limited to a single reflective exercise. Instead, metacognition was embedded from the beginning to the end of instruction: before solving through task identification and planning, during solving through monitoring and adjustment, and after solving through evaluation and reflection.

Validation-Informed Refinements

The expert review strengthened the instructional design in three main ways. First, the language of learner prompts was simplified. Initial metacognitive prompts were revised so that Grade 6 learners could understand and respond to them without needing technical knowledge of metacognition. This refinement helped preserve the theoretical intent of the design while making the activities developmentally appropriate.

ISSN (Online): 2455-3662
EPRA International Journal of Multidisciplinary Research (IJMR) - Peer Reviewed Journal
Volume: 12| Issue: 5| May 2026|| Journal DOI: 10.36713/epra2013 ||SJIF Impact Factor 2026: 8.753 || ISI Value: 1.188

Second, the flow of activities was adjusted to reduce cognitive load. The final sequence moved from visual recognition of shapes to problem classification, guided formula application, hands-on measurement, independent problem-solving, and reflection. This progression allowed learners to build from concrete identification toward more abstract and evaluative problem-solving processes.

Third, the teacher's facilitation role was clarified. The design emphasized guided questioning, midway checking, learner verbalization, and reflective closure rather than extended teacher explanation alone. This refinement made the lesson more consistent with the goal of making learners' thinking visible and regulated during mathematical problem-solving.

DISCUSSION

This study developed a theory-based metacognitive instructional design for Grade 6 mathematical problem-solving. The resulting design demonstrates how Divinagracia's Metacognitive Theory can be translated into classroom-based instructional activities that are observable, developmentally appropriate, and aligned with a specific mathematics competency. Rather than presenting metacognition as an abstract construct, the design operationalizes it through concrete lesson phases, learner actions, and metacognitive indicators.

A major contribution of the design is its domain-specific orientation. Metacognitive instruction is more meaningful when it is connected to the actual cognitive demands of a learning task. In this study, the design was embedded in composite-area problem-solving, a topic that requires learners to recognize visual structures, decompose composite figures, recall area formulas, select solution strategies, and evaluate answers. This connection between metacognition and mathematical content addresses the concern that metacognitive skills may not automatically transfer across domains without deliberate instructional support (Desoete et al., 2003). By situating metacognitive processes within a specific Grade 6 geometry and measurement competency, the design provides a clearer pathway for teaching learners how to regulate their thinking while solving actual mathematics problems.

The design also extends the usual treatment of metacognition beyond the familiar planning-monitoring-evaluation cycle. Although these three processes remain essential, Divinagracia's theory enabled the instructional design to include additional dimensions of problem-solving experience. Learners were guided to identify problem typology, analyze the nature of mathematical problems, become aware of relevant mathematical knowledge, recognize personal strengths and uncertainties, engage in bodily motion experiences, reflect on emotions and attitudes, and evaluate solution quality. This broader treatment is important because learners' struggles in mathematical problem-solving may arise from different sources. Some learners may fail to recognize the problem type, some may misunderstand the task structure, some may lack confidence, some may become discouraged by difficulty, and others may need concrete experiences before abstract reasoning becomes meaningful.

The inclusion of problem typology and problem nature at the beginning of the lesson is particularly significant. Many learners tend to begin solving immediately after reading a problem, often applying a familiar formula without first examining whether the strategy fits the situation. The developed design deliberately positioned task identification and problem framing before computation. Through shape recognition, picture analysis, and discussion of routine and non-routine problems, learners were guided to understand the problem before choosing a solution path. This sequence supports strategic rather than mechanical problem-solving and is especially relevant for non-routine tasks, where direct procedural recall may be insufficient.

Another important feature of the design is its emphasis on making learner thinking visible. Each metacognitive domain was linked to observable classroom behavior. Planning became visible when learners identified problem types and wrote solution plans. Monitoring became visible when learners paused to check progress or explained their reasoning. Regulation became visible when learners adjusted strategies or sought clarification. Embodied cognition became visible when learners measured, traced, pointed to component figures, or physically represented parts of a composite shape. Affective regulation became visible when learners reflected on accuracy, persistence, and responsibility. Evaluation became visible when learners justified answers, revised errors, and assessed whether their solutions were reasonable. These indicators can help teachers recognize and support metacognitive development during instruction.

The design's inclusion of bodily motion experiences is also noteworthy. Elementary mathematics often benefits from concrete and physical engagement, particularly when learners work with geometric concepts. By incorporating measurement, tracing, pointing, and decomposition of figures, the design connected physical action with abstract mathematical reasoning. This feature reflects the embodied dimension of Divinagracia's theory and supports the idea that learners' movements and visual-spatial interactions can contribute to mathematical understanding. In composite-area tasks, bodily motion can help learners see how a figure is divided, where dimensions apply, and how partial areas contribute to the whole.

The affective dimension of the design further strengthens its instructional relevance. Mathematical problem-solving can evoke uncertainty, anxiety, frustration, or persistence. By including a valuing activity anchored on real-life applications such as tile installation, the design encouraged learners to connect mathematical accuracy with responsibility and practical consequences. This component recognizes that learners' attitudes toward problem-solving influence how they sustain effort, respond to errors, and value careful reasoning. Such an emphasis is consistent with research highlighting the relationship between affective regulation and mathematical engagement (Di Leo & Muis, 2020; Wang et al., 2021).

The design also contributes to instructional practice by showing how theory-based instruction can remain feasible within regular classroom conditions. Expert review led to revisions that simplified metacognitive prompts, improved time allocation, reduced cognitive load, and embedded metacognition through guided questioning rather than rigid structures. This refinement is important because theoretically rich instructional designs may fail in practice if they are too complex for learners or too difficult for teachers to implement. The final design balanced theoretical alignment with classroom feasibility, making it more suitable for Grade 6 learners and for implementation by a classroom teacher.

Overall, the developed instructional design offers a structured model for embedding metacognition across the full mathematics lesson sequence. It begins with task recognition and problem framing, proceeds to strategic application and monitoring, incorporates embodied and affective engagement, and ends with evaluation and reflection. This progression reflects the view that mathematical problem-solving is not merely a sequence of computations but a regulated process of understanding, strategy use, adjustment, and justification. As a design output, the model provides a basis for future intervention studies that may examine its effects on learners' problem-solving performance, metacognitive strategy use, and engagement with routine and non-routine mathematical tasks.

Conclusion

This study developed a theory-based metacognitive instructional design for Grade 6 mathematical problem-solving. Grounded in Divinagracia's Metacognitive Theory, the design translated seven metacognitive domains into classroom activities for composite-area problem-solving. The resulting instructional sequence integrated shape recognition, problem classification, formula application, guided self-questioning, hands-on measurement, valuing, and reflective evaluation.

The design contributes to mathematics education by operationalizing metacognition as a set of observable and teachable classroom practices. It embeds metacognitive processes across the full lesson sequence, from task identification and planning to monitoring, affective regulation, embodied engagement, and solution evaluation. As a design output, it provides a context-sensitive model for helping Grade 6 learners engage more strategically and reflectively in mathematical problem-solving.

REFERENCES

1. *Brown, T. (2008). Design thinking. Harvard Business Review, 86(6), 84–92.*
2. *Desoete, A., Roeyers, H., & De Clercq, A. (2003). Can offline metacognition enhance mathematical problem solving? Journal of Educational Psychology, 95(1), 188–200. https://doi.org/10.1037/0022-0663.95.1.188*
3. *Di Leo, I., & Muis, K. R. (2020). Confused, now what? A cognitive-emotional strategy training intervention for elementary students during mathematics problem solving. Contemporary Educational Psychology, 62, Article 101879. https://doi.org/10.1016/j.cedpsych.2020.101879*
4. *Divinagracia, L. S. (2018). Metacognitive processes in successful mathematical problem solving. Future Ed Special Edition: Multidisciplinary Research Journal, 5(1), 70–86.*
5. *Divinagracia, L. S. (2022). A systematic grounded theory on how metacognition facilitates mathematical problem-solving. Journal of Higher Education Research, 10(1), 30–65.*
6. *Flavell, J. H. (1979). Metacognition and cognitive monitoring: A new area of cognitive-developmental inquiry. American Psychologist, 34(10), 906–911. https://doi.org/10.1037/0003-066X.34.10.906*
7. *Jacobse, A. E., & Harskamp, E. G. (2009). Student-controlled metacognitive training for solving word problems in primary school mathematics. Educational Research and Evaluation, 15(5), 447–463. https://doi.org/10.1080/13803610903444519*
8. *Lee, N. H., Yeo, D. J. S., & Hong, S. E. (2014). A metacognitive-based instruction for Primary Four students to approach non-routine mathematical word problems. ZDM Mathematics Education, 46(3), 465–480. https://doi.org/10.1007/s11858-014-0599-6*
9. *Okumuş, Ö., & Öztürk, M. (2024). Metacognitive training for algebra teaching to high school students: An action research study. Learning and Motivation, 88, Article 102064. https://doi.org/10.1016/j.lmot.2024.102064*
10. *Pande, M., & Bharathi, S. V. (2020). Theoretical foundations of design thinking: A constructivism learning approach to design thinking. Thinking Skills and Creativity, 36, Article 100637. https://doi.org/10.1016/j.tsc.2020.100637*
11. *Schneider, W., & Artelt, C. (2010). Metacognition and mathematics education. ZDM Mathematics Education, 42(2), 149–161.*
12. *van den Akker, J., Gravemeijer, K., McKenney, S., & Nieveen, N. (Eds.). (2006). Educational design research. Routledge.*